CHAPTER 5

Mathematics Teaching for Understanding: Reasoning, Reading, and Formative Assessment

Paula Miller and Dagmar Koesling

IN THIS CHAPTER: Paula Miller and Dagmar Koesling describe the role that literacy plays in three central elements of Koesling's mathematics instruction: teaching for understanding by solving complex word problems, reading mathematical text, and assessing student understanding. Readers see how literacy helps all students, including English language learners, "marry" skills and understanding.

KEY POINTS

- Students, even those with weak math skills, can develop problem-solving and reasoning skills that make advanced mathematics courses accessible.
- Teachers can "marry" the mathematical reasoning process with a reading process that helps students understand the real-world context and mathematical concepts of a problem.
- Rich word problems provide the teacher with in-depth assessment of a student's skills, which in turn helps teachers determine what additional instruction students need.

Delia Jones, a student at Boston's Fenway High School, represents many students who make it through middle school still lacking basic math skills. Unlike most of those peers, however, Delia is now a senior precalculus student—and doing well. Asked about her school experiences in mathematics, Delia recalls:

> When I came into the ninth grade, I didn't even know how to do long division. My teachers in elementary and middle school would . . . show us how they'd work a problem. It felt . . . like watching a magician pull a rabbit out of a hat. "See how easy this is? Now you do it, too." But the problem was, I never figured out why the rabbit got into the hat, much less what steps the magician used to make it reappear.Then the teachers would give

> us . . . [similar] problems to do in class and finish as homework. I'd try, but then I'd give up. The teacher made it look so easy, but I felt stupid when I couldn't do it, so I just started hiding.

Delia admits, "It was easy to hide in middle school. You know, you just keep quiet, keep your head down . . . don't cause trouble. The teachers didn't know how lost I was until I'd already failed the test. Then it was too late and they were already moving onto the next thing," Delia says. "But here at Fenway, teachers figure out what I don't know even before *I* know I don't know it!"

ACCESS FOR ALL: THINK IT THROUGH VERSUS DRILL IT IN

Fenway High School is a small high school (within the Boston Public School system) with approximately 300 students, 65% of whom qualify for free or reduced-price lunch, and 85% of whom are people of color (although few formally qualify as English language learners).

Teachers at Fenway believe that problem-solving and reasoning skills, as well as content skills, can be taught best in the context of a literacy-based math curriculum. Such an approach goes beyond drills and memorized procedures. It not only equips students with the skills to pull mathematical rabbits out of hats, but also challenges students to understand how those rabbits got into the hats in the first place.

Dagmar Koesling teaches math at Fenway and works as a math consultant with the Public Education & Business Coalition, where Paula Miller is a staff developer. In this chapter, we describe three central components of Dagmar's mathematics instruction that help her make advanced mathematics courses accessible to all students:

1. Using complex word problems to teach higher level math skills by giving students a concrete real-world situation and applying progressively more abstract algebra tools
2. Modeling the mathematical reading and reasoning process, coaching students as they learn difficult concepts
3. Assessing and naming students' thinking as they work, and reshaping teachers' next steps accordingly

MATH LITERACY AS A CIVIL RIGHT

Advanced math courses are a gatekeeper. In order to receive a high school diploma, Dagmar's students must pass the high-stakes tests of the Massachusetts Comprehensive Assessment System (MCAS), which require mastery of algebra, geometry, and process skills. And beyond these basic graduation requirements, students need to master higher level courses such as calculus, or the door to careers in science, economics, and technology will be closed to them.

Many students enter high school either having never mastered mathematics or not retaining that mastery:

- Students may have experienced math as isolated skills and formulas leading to solutions that make little sense.

- Students who "live in poverty, students who are not native speakers of English, students with disabilities, females, and many non-white students . . . [often are the] victims of low expectations" (National Council of Teachers of Mathematics, 2000, p. 13).

Dagmar believes that "to level the playing field for my students, I must facilitate the learning of higher level math like algebra and process skills like problem-solving and reasoning to kids who have been judged by others as not having the aptitude to achieve at that level. Learning algebra and process skills is a civil right."

TEACHING ALGEBRA SKILLS THROUGH COMPLEX WORD PROBLEMS

Dagmar and her colleagues use the Interactive Mathematics Program, a math curriculum that challenges students to learn math concepts through complex, language-based problems. This approach is supported by leading math educators; Arthur Hyde (2007) asserts: "To raise mathematics achievement in the United States to higher levels it is essential that we infuse language and thought into mathematics. We can do a far better job of teaching students to understand and love mathematics if we enrich our teaching with practices from reading and language arts adapted by cognitive science" (p. 48).

"Leading students toward building their own understanding of a math concept takes time," says Dagmar. "I could give students the answer to the problem much earlier in the game . . . but students would have lost a lot of learning." Instead, as we show in this chapter, Dagmar may spend the better part of a class period helping students deconstruct and solve one word problem.

The results are impressive: Though 65% of Fenway High School students qualify for free or reduced-price lunch, the school boasts a 99% graduation rate, and 84% of graduates enroll in colleges. And even though half of the entering ninth graders were deficient in math and reading skills, 91% of sophomores met or exceeded state mathematics proficiency standards as measured by the state MCAS test on the first try, and 100% met or exceeded profiency on the English MCAS.

These successes, as measured by statistics, result in part from the teachers' use of a three-pronged process: using complex word problems, coaching students on how to read math problems, and assessing students' thinking in order to revise instruction.

REASONING AND READING LIKE A MATHEMATICIAN

Like most secondary math teachers, Dagmar received no coursework in reading during her formal teacher training. Yet she has made it her business to include reading in her pedagogical toolkit. In this chapter, you will see her challenge students to develop algebra skills through word problems by teaching them a mathematical reading process (see Figure 5.1).

"I have students read the problem at least twice before they actually try to solve it," Dagmar says. "This gently guides students through their thinking about the text. I ask open-ended questions to further understanding and uncover conceptual

Figure 5.1. Mathematical Reading and Reasoning Process

First Read: Read for Understanding

- What vocabulary do I not know?
- What's the real-world context of the problem?
- What's the situational setting of the problem (not the mathematical skills)?
- What questions are being asked?

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Second Read: Identify a Problem-Solving Process

- What is the pertinent information in this problem?
- What problem-solving strategies could I use?
- Which of those problem-solving strategies is best suited for this problem?
- How will I represent the problem in the symbolic language of mathematics?
- What mathematical details will I select as I reason and solve this problem?

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Third Read: Solve the Problem and Check for Reasonableness

- Now that I understand the problem's content, how can I best use my math skills to solve the problem?
- How can looping back to the original setting help me interpret the solution?

misunderstandings. It's not 'here's how you do it.' It comes from breaking the problem-solving process into small pieces that the student is able to successfully negotiate."

Students watch her model as they read the problem together three times, in specific ways that support mathematical reasoning:

First read. Reading for understanding: What is the real-world setting of the problem?
Second read. Identifying a problem-solving process.
Third read. Solving the problem and checking for reasonableness.

Dagmar also uses these multiple reads to assess students' conceptual understanding, reasoning, and communication (Cai, Lane, & Jakabcsin, 1996). Below, we show Dagmar taking students through these three reads and describe some of her assessment strategies. For the second read—perhaps the most complex—we also describe how Dagmar provides additional support to English language learners. At the end of the chapter, we explain Dagmar's assessment strategies in more detail.

THE FIRST READ: READING FOR UNDERSTANDING

Dagmar asks a student to read the problem aloud to the whole class:

> A chemical company spends $2 million to buy machinery before it starts producing chemicals. Then it spends $0.5 million on raw materials for each million liters of chemical produced.
>
> 1. The number of liters produced ranges from 0 to 5 million. Make a table showing the relationship between the number of million liters produced, *L*, and the total cost, *C*, in millions of dollars to produce that number of million liters.
> 2. Find a formula that expresses *C* as a function of *L*.

Note: From *Functions Modeling Change: A Preparation for Calculus* (p. 10), by E. Connally, Deborah Hughes-Hallett, and A. M. Gleason, 2000, New York: Wiley.

Clarifying Vocabulary

Dagmar knows that students must understand the key words before they can understand the full problem. So she asks, "What is a liter?" Her question meets blank stares. She considers two choices: She could tell students, "There are roughly 3.5 liters in a gallon," or she could show students a visual image of a liter to help them make connections to their background knowledge. She quickly pulls a liter Coke bottle from the recycle bin, holds it up, and says, "This is what a liter looks like. Pass it around so you can make the kinesthetic connection between a bottle and a liter."

Comprehending the Context

Dagmar wants students to take ownership of the problem and rephrase it so it makes sense to them. "Who can tell me in their own words what the problem is about?" Dagmar asks the class. Xiomara responds by saying, "A company makes chemicals and it costs money." This is a good summary of the big picture.

When a student shares such a summary, all students benefit: the repetition reinforces comprehension, and students have a chance to ask clarifying questions.

Stating the Problem's Questions or Tasks

In the first reading Dagmar makes sure students can state the questions or tasks of the problem. If students can't, then Dagmar doubles back to give more direct instruction around vocabulary or the problem's setting. This immediate remediation reduces potential student confusion.

In this lesson the class struggles to correctly state the questions. Xiomara got the big picture, but she did not state that the problem asked her to make a table and set up a function. This shows Dagmar that Xiomara needs more instruction on how to read to identify the questions or tasks.

Dagmar then gives that instruction to the class. "When you read an essay, you have a topic sentence and supporting ideas. The most important idea is generally at the beginning. But in a math problem, you read the information first, and the most important part, the question or task, is at the very end. That's why you must read a math problem several times. It's important to ask yourself not only 'What is this problem about?' but also 'What are we supposed to do?' Look at the problem again and try that out. Look at the very end to see if you can find the question or task to be solved."

ASSESSING THE FIRST READ: STUDENTS' UNDERSTANDING OF THE REAL-WORLD CONTEXT

It is important to assess whether students have mastered the goals of the first read. If they do not understand the setting, vocabulary, or meaning of the questions or tasks to be solved, they will not be able to continue the problem.

To assess, Dagmar sometimes asks students to write down the problem questions they have identified, or she may ask students to state the questions verbally to the class. Today she pulls a "dipping stick"—a wooden tongue depressor inscribed with a student's name—from a cup, and asks, "Chris, now that you've reread the problem and focused on the last part, what does this chemical company want to know?"

"This chemical company wants to figure out the actual cost of making stuff," Chris says.

"That's it!" Dagmar responds. "And what helped you determine that this is the question that needs to be answered?"

Another student raises her hand. "That thing you said about the important part being at the end of the problem—that helped us out a lot. Before we were analyzing every sentence looking for the question."

Students have accomplished two parts of the problem-solving process: They understand the setting of the problem, and they have identified the questions to be answered. During this first read and the related assessment, Dagmar gave her students two literacy gifts: a process to follow, and an opportunity to reflect on how that process helped them accomplish the task.

THE SECOND READ: CHOOSING A PROBLEM-SOLVING STRATEGY

Without rereading the problem, the math problem-solving process would grind to a halt. The second read is more complex than the first and actually incorporates multiple, increasingly precise rereads. Although a skilled reader might be able to perform all these tasks during and after one second read, Dagmar addresses each part separately in order to build students' problem-solving skills.

Looking for Pertinent Information

"If Chris is on target, that the question this problem asks is what's the actual cost of making products for the chemical company, then what do we have to do with this problem? Reread the problem and underline phrases and sentences that you think may have important information for us. Find data that could help the company determine how much it will actually cost to make their new product. We may not know exactly how to use that information yet, but we have a hunch it will be useful. Talk it over with your neighbor."

Dagmar lets students read the problem again and gives them a minute to chat with each other.

She wants to make the process of identifying pertinent information explicit to all students. Circulating as students work, she sees that Maurice has underlined several pieces of information, and asks him to share with the class. "So, Maurice, what pieces of information did you find that will help us determine the cost to produce?" Dagmar asks.

"This company has spent two million bucks on equipment before they've even made a dime," Maurice says.

"So what questions were you asking yourself, Maurice?"

"Right here it says the company spent $2 million to buy machinery before producing chemicals. So I wondered if that should be included when the company adds up all the costs. Won't they need to add that $2 million to the list?" Maurice says. He is modeling for his peers how to think through what is most important.

Determining Problem-Solving Strategies

Students next need to consider possible strategies. Dagmar helps students do this by first labeling Maurice's thinking for the class. "Maurice took a piece of information from the text itself, that the company paid $2 million up front for equipment. He asked himself a question, 'Shouldn't that be included in the total costs for the new product?' And that question is worth considering as he thinks about the problem-solving strategy he'll use."

She asks Maurice, "So what math operation would you use to make sure the chemical company includes that $2 million in their final costs?"

"You need to add up all of the company's expenses first thing. And the cost of equipment is one of those expenses," he responds.

Dagmar affirms his response, then questions the class, "So what other information do you think is important here? What are some of the other costs we'll need to include?"

Another student responds, "They spend $0.5 million for each million liters produced."

The class now has correctly identified all the pertinent information in the problem and one math operation to use in solving it.

Representing the Problem in Mathematical Symbolic Language

The next part of the second read is perhaps the hardest: Students need to move from the concrete, real-world situation to representing it in abstract mathematical symbols.

In this case, students need to make a table for 0 to 5 million liters of chemical produced. From experience, Dagmar knows that not all students will be able to do this immediately.

Again, she makes the reasoning and reading explicit. She asks them what the problem asks them to do next. Noticing that few students have their eyes on the text, Dagmar redirects the class. "Look at the problem again."

She knows that students tend to read only the main part of the problem, not the questions or tasks at the end. "When you fail to read the question and tasks at the end of the problem," Dagmar explains to the students "it's like telling a joke without the punch line."

Students read the question quietly to themselves. Some hands go up. Dagmar waits until all students look up, an indicator that they are finished reading.

ASSESSING THE SECOND READ: STUDENTS' ABILITY TO ABSTRACT

As with the first read, Dagmar begins to assess how well students are mastering the goals of the second read.

She draws a dipping stick with Maria's name. "What do you think, Maria?"

Maria answers, "Make a table."

"Right," Dagmar responds. "Question 1 does give us the problem-solving strategy of 'make a table.' And it also provides specifics about the table. What might those be?"

Students now need to represent the problem in mathematical symbolic language. They must transition from the concrete problem situation (i.e., the cost of producing chemicals) to the abstract algebra concepts of dependent and independent quantities. They also have to grapple with the abstract concept of variable, correctly identifying the variables *L* and *C* and their precise meaning in terms of the problem. And since the text is not explicit about the format of the table, students must draw on their algebra skills and a correct interpretation of the facts of the problem to set up the table.

While students read the text they must ask themselves the key algebra question for the entire problem: Which quantity depends on which? Adults may easily see that cost depends on volume of chemicals produced. But that is not obvious to many of Dagmar's students.

Visual Representations: Drawing a Table

To help students make sense of the abstract concepts in the problem they just read, she draws a blank table on the board.

Through open-ended questions, she gets students to label the table correctly. She asks, "What are the variables mentioned in the text?"

Students respond, "*L* and *C*."

"What does *L* stand for?" Dagmar asks the class.

Students call out, "Liters."

Dagmar continues to push for more information. "Is that all I should put in the table? How can we make this more specific?"

"Number of millions of liters of chemical produced," Jasmine says, and Dagmar adds that clarification to the table.

"And what's the other variable?" she asks.

"It's *C*," several respond.

"So what does *C* represent with units?" she asks.

"Cost in million of dollars," they reply.

Dagmar pulls from the students the information they are finding in the text, and writes their words on the board.

She has now modeled how to find the variables and correctly identify each one in terms of the problem setting. Next, she quickly thinks aloud about how she processes pieces of this word problem.

"Here's what I ask myself when setting up a table, 'What depends on what? Does the amount of chemical depend on the final cost or does the final cost depend on the chemical produced? Which quantity goes on the left side of the table and which on the right side?"

Shifting the thinking responsibility back to students, she asks: "What do you think?"

"*L* belongs on the left and *C* on the right," James responds. Dagmar enters the variables in the top row.

Checking for Misconceptions

She quickly assesses the class by asking the class who agreed with James. Most students raise their hands, but not everyone. This indicates that some did not make the conceptual leap to the dependent and independent variable.

Instead of delving into an abstract conversation about independent and dependent variables Dagmar decides to go back to the text and keep the conversation at the concrete level.

"Look at the question one more time," she tells the class. "Where are the numbers we care about right now?" She waits ten seconds.

Assessing Students' Understanding of Technical Vocabulary

"Now we have this table that's begging to be filled in with numbers. What do we call these numbers? They have a special name," Dagmar tells the class as she points to the Word Wall (see Figure 5.2) loaded with math terms students have learned.

Figure 5.2. Mathematical Word Wall

Independent Variable	Dependent Variable
X	*Y*
Input	Output
Domain	Range
Horizontal axis	Vertical axis
Input value	Function value

"Oh! You mean independent variable," cries out Monica.

"Right!" Dagmar said. As she points to the first empty spot in the table, she asks, "What goes here?"

Monica responds, "Zero."

"Why do you think that?" Dagmar quizzes.

Monica looks at the text again, then responds, "The left column holds the independent variable *L* because these are the input numbers. Whatever happens in the right column *depends on* the number in the left column. That would make the right column the dependent variable. The second question reads, 'Find a formula that expresses *C* as a function of *L*.' I think this means that *C* is the dependent variable."

Dagmar knows that Monica understands the most important concepts of the abstract algebra component of the problem: dependent and independent variables and where to put them in the table. She checks that this assessment is correct by asking, "So what do I write here on the left side of the table, Monica?"

Monica quickly rattles off, "0, 1, 2, 3, 4, 5 because that's what it tells us in the first line of the problem."

Monica has achieved the main goal of the second read: She has translated the real-world context of the problem into a mathematical representation (i.e., a table).

THE SECOND READ: ADDITIONAL SUPPORT FOR ENGLISH LANGUAGE LEARNERS

During the second read, Dagmar often provides additional support to her English language learners and others who may need more scaffolding. "For an ELL, the targets are (a) to stretch the student towards conceptual learning of the content at the next level and (b) to bring her or his language development forward at the same time" (Bay-Williams & Herrera, 2007, p. 49).

Dagmar knows that Charlie, an ELL student, often needs extra time and support to fully understand a problem. She does not want to lose track of Charlie or let him off the hook; he too can master reading and math.

To assess his current understanding, Dagmar points to the empty spot on the table across from where $L = 0$, and asks Charlie, "What goes here?"

Charlie responds, "$C = 0.5$."

This is incorrect, yet Dagmar does not correct him. She wants to give him the challenge of thinking this through.

Additional Processing Time for ELL Students

Charlie may have chosen the wrong number because he did not fully comprehend the problem. He may simply need more processing time.

"Read the beginning of the problem again, Charlie. What does it say?" In effect, Charlie gets a chance to repeat the second read.

As an English language learner, Charlie may need this extra time in order to "translate" the math into his native language. As ELA expert Sally Nathenson-Mejia has noted, math is a language with a specific syntax; typically, whatever language students *first* used when learning to calculate math is the language they will revert to whenever they calculate math (personal communication, December 15, 2007).

Chunking Text for ELL Students

Charlie reads the first sentence aloud. Dagmar asks him to stop there. She doesn't want him to read the information in the following sentence because that isn't pertinent to understanding what needs to go into the table. Thus Dagmar breaks the reading down into small chunks for Charlie. As an unskilled math reader, Charlie might jumble up additional information and get confused.

Clarifying Vocabulary and Wait Time for ELL Students

Dagmar knows that English language learners often struggle with technical math vocabulary. She asks Charlie directly if he needs clarification on any vocabulary. He says he is fine.

She waits as he continues to look at the first sentence for an answer. After a few seconds, she can tell by his body language—a smile on his face and his body physically relaxing—that he is able to do this. She confirms this when Charlie looks up and says, "Oh. It's 2."

"That's exactly right. And what in the text helped you figure that out?" she inquires.

Charlie looks at the ordered pair $L = 0$ and $C = 2$ and says, "Because in the beginning of the word problem, it says before they make anything, it cost two million dollars."

Dagmar's assessment helped her adjust her instruction so Charlie arrives at the right answer on his own, giving him the opportunity to learn and gain confidence.

Naming Success for ELL Students

Dagmar points to the next empty spot on the table and asks, "So try that out, Charlie. Look at the next number in the *L* column of our table and tell me what goes on the right side."

"2.5!" Charlie exclaims.

"And the next spot?"

"3!" he says.

With a big smile on his face, Charlie continues to fill in the rest of the missing numbers (see Figure 5.3).

"So Charlie, why is that? What's happening here?"

"Well, there's a pattern. It goes up each time by 0.5 million dollars," he says.

Students have read the problem twice, identified the setting and the questions, translated the problem into mathematical representation, and made a table. They now are now ready to set up the function that models the problem.

THE THIRD READ: INTERPRETATION OF THE SOLUTION

The most abstract part of solving complex word problems is translating the concrete word problem into an algebraic equation, or function, in this example, $C(L)= 0.5\,L + 2$.

Figure 5.3. In-Out Table for Cost of Chemicals Produced Depending on the Number of Liters Produced

L Liters of chemicals produced (in millions)	*C* Cost (in millions of dollars)
0	2.0
1	2.5
2	3.0
3	3.5
4	4.0
5	4.5

By laboring through the table, students have had to think about the pattern involved, which makes it easier to set up the equation. They are able to identify the 2 as the initial cost of $2 million and the 0.5 as the variable cost of $0.5 million per million liters produced. Both contribute to the total cost *C*.

After all this work many students don't remember anymore what they were supposed to do and what it all means. So it is important to have students verbally summarize and interpret their work. Students need this final processing opportunity to help them solidify the skills learned and put it all in its proper context.

In this problem, students were to make a table and set up a function to model the cost of producing 0 to 5 million liters of chemicals. After Dagmar writes the cost equation on the board she asks the class about the meaning of the equation. As she points to the equation, she asks, "Who can tell me what each part of the equation represents?"

Students look at the problem one more time as they struggle to make sense of the equation. The answers come one by one and Dagmar asks that students label the equation as she writes on the board (see Figure 5.4). She wants students to interpret the equation on both a concrete and an abstract level.

Only now can Dagmar be confident that students have mastered all aspects of the problem: text, table, equation, and understanding.

These three reads named students' mathematical reasoning processes so everyone could begin to see how to get the rabbit out of the hat.

ASSESSMENT LEADS TO STUDENT EMPOWERMENT

Dagmar used informal assessments throughout all three reads in order to understand student progress. These informal assessments let her look at students' mathematical reasoning and reading and determine whether they are on target or not.

Figure 5.4. Interpreting All Parts of the Equation

C	**(*L*)**	**= 0.5**	***L***	**+ 2**
C	*L*	0.5	*L*	+ 2
Total cost (Dependent variable)	Mill. liters of chemicals produced (Independent variable)	Expense of \$0.5 mill. for each mill. liters produced (Variable cost or rate of change)	Mill. liters of chemicals produced (Independent variable)	\$2 mill. for equipment before production starts (Fixed cost or initial condition)

Note: Typically, this equation is written as follows: $C(L) = 0.5L + 2$. When Dagmar teaches this, students frequently do not understand what all the symbols in the equation mean. Therefore, she spaces out the equation and explain the real-world meaning and the mathematical meaning of each symbol.

She then can adjust her next instructional moves accordingly to prevent the cycle of learning shut-down that plagues many students.

As shown in Figure 5.5, informal assessments take many forms. Dagmar used many of these informal assessments as students worked through the chemical company word problem. She consistently questioned students about their thinking to assess when their conceptions were accurate or off-target.

Using the dipping sticks helped insure that she talked with many students versus only those who raised their hands. Having students write gave accountability to each student and also helped them process their thinking to share with partners.

The thumbs check allowed her to quickly determine how many students needed more teaching and how many were ready to move ahead. For those needing more teaching, she conferred with them individually or in small groups. For those ready to move ahead, she challenged them to fill in the table on their own before sharing with the class.

SKILLS AND UNDERSTANDING GO HAND IN HAND

The guiding focus for Dagmar's math instruction is her belief that students must do the thinking and explain why they select specific math processes to work toward solutions. When students construct knowledge, it becomes more enduring knowledge. Literacy becomes the tool that lets them access and assess math concepts, bringing together formerly disparate skills and approaches to solving problems into a coherent whole. Literacy provides students a key—especially important for those who might otherwise be locked out—to mathematical understanding and skill, which in turn empowers them to open many doors.

Figure 5.5. Informal Assessment Methods in Literacy-Rich Math Instruction

INFORMAL ASSESSMENT METHOD	HOW OR WHEN TO USE IT
Questioning	Use throughout each of the three math reads to help students clarify their thinking and identify misconceptions
Conferring	Meet with each student or small groups to query their thinking as they work on a math task: • Where are you stuck? • What have you tried to get unstuck? • What are you considering as your next move? • What is the question asking of you?
Dipping sticks	Randomize whom you call on by using wooden tongue depressors with a student's name on each stick and pulling them from a container, grab bag style, to involve everyone in class conversations
Thumbs check	Students give the following signals to help the teacher assess their understanding during class: • Thumbs up: "I understand the concept." • Thumbs sideways: "I sort of get it." • Thumbs down: "I need you to teach a bit more."
Exit slips	Students take the last 5 minutes of class to write three or four sentences that: • Explain the main point of the lesson (e.g., what a function is) • Name one strategy used in class that helped the student solve a problem
Journal writing	Writing daily to short prompts, such as: • Explain this answer • Explain how you solved this problem

HOW TO BEGIN

- Experiment with the 3-part math reading process.
- Let students wrestle with word problems. Don't rescue.
- Coach students to articulate their thinking, and to ask themselves these questions:

Do I really understand this problem?
What process should I use to solve this problem?
What skills do I need to use?
What does the answer mean?
How can I communicate this to others?

LINGERING QUESTIONS

- What must mathematics teachers give up to find time to teach this mathematical reading process? What will students gain?
- How can content area teachers in different disciplines collaborate to understand the differences students face as readers in various content areas?
- How might such conversations help teachers to help students access text across all disciplines?

LEADERSHIP PERSPECTIVES

From Diane Lauer, former principal of Conrad Ball Middle School:

- *Importance of scaffolding.* We tend to think that older students don't need us to break things down for them. But Koesling shows the benefits of scaffolding the type of reading and reasoning she values most for her high school math students. She models, coaches, gives them time to practice, and asks them to reflect on the tools they are learning. She explicitly teaches students how to think mathematically. Consider to what extent you do, could, or should do this for your students in your content.
- *Leaders must walk the talk.* School leaders must ask themselves, "Am I, as a principal, doing the things Koesling does to help students learn when I help my teachers learn? What do I model? What scaffolds or supports do I provide? When and how do I give teachers time and space to practice new skills? When and how do I bring my staff together to reflect on new learning?"

From Garrett Phelan, principal of César Chávez Charter School (Capitol Hill campus):

- *Value of rereading.* I encourage my teachers to have students read the same text multiple times. Rereading with literacy strategies does more than help a student understand the given text; it creates a lifelong habit in students of probing, asking more of themselves and of the texts they encounter. Thus it shifts the responsibility for learning back onto the student.
- *Informal assessment.* Miller and Koesling say, "When students construct knowledge, it becomes more enduring knowledge." Informal assessments—conferring, seminars, reflection slips, writing prompts, and so on—help individual students construct knowledge. They have to do the thinking and explaining of the process they are using to solve math problems or problems in any discipline.
- *Access for all.* We can make advanced classes accessible for all students if we create the conditions for success, honor student thinking, and give students literacy

skills. Consider how shifting instructional practices in math might mean giving more access to more students at your school.

RELATED READINGS

Arthur Hyde's book *Comprehending Math* (Hyde, 2006) explains step-by-step how students can use reading and thinking strategies to approach word problems.

Lainie Schuster and Nancy Anderson's book *Good Questions for Math Teaching* (Schuster & Anderson, 2005) gives models of how to ask purposeful questions that build student understanding.

Douglas Fisher and Nancy Frey's book *Checking for Understanding* (Fisher & Frey, 2007) provides various engaging activities to help build and assess student understanding.

Jan de Lange's essay "Mathematics for Literacy" (de Lange, 2003) discusses different forms and definitions of math literacy.

Joan Kenney's book *Literacy Strategies for Improving Mathematics Instruction* (Kenney, 2005) is full of concrete examples and advice on how to teach math literacy skills.